Domain & Range of Functions

➢ The **domain** of a function is the set of all the numbers you can substitute into the function (x-values).

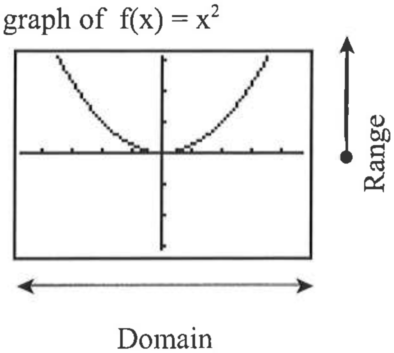
➢ The **range** of a function is the set of all the numbers you can get out of the function (y-values).

For example: $f(x) = x^2$

What's the domain? Well, you can substitute any real number you want into this function: You can square 4, ½ , -7, 1.01738, or whatever, and you get an answer. So the domain is all real numbers. In interval notation this is written $(-\infty, \infty)$.

What's the range? Well, let's think about it. If you substitute any number into this function, are you ever going to be able to get a negative number out of it? Nope! So it looks like the range of this function is the set of all non-negative numbers (the positive numbers plus zero). In interval notation this is written $[0, \infty)$.

Now look at the graph of $f(x) = x^2$



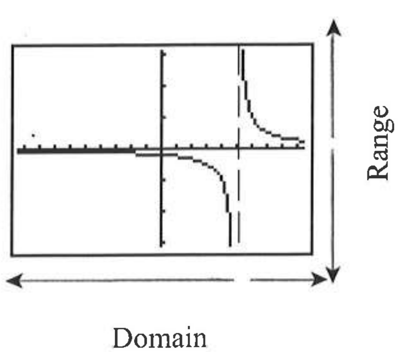
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Let's try another example: $f(x) = \dfrac{1}{x - 5}$

What numbers can we substitute into this function? Well, it looks like any number will do. But wait! What happens if we substitute in $x = 5$? We divide by zero, which is not allowed. So let's not substitute in 5. Since any other number is fine, the domain of this function is: all real numbers except 5. In interval notation this is written $(-\infty, 5) \cup (5, \infty)$.

Range is sometimes harder to figure out from a formula. This is where the graph comes in handy.



Domain: $(-\infty, 5) \cup (5, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$ [since the graph never touches the x-axis]

When determining the domain of a function from a formula, we really only have to look out for two situations:

(1) **Rational Expressions** (fractions) - Division by zero is not allowed so we must omit any values of x which make the denominator zero.

(2) **Even Roots** - For even roots such as square roots, the radicand can not be negative (the square root of a negative number is not defined as a real number). In this situation we must make sure the radicand is non-negative (that is, greater than or equal to zero).

<u>**Examples**</u> Find the domain of each of the following functions

(A) $f(x) = \dfrac{5x}{x^2 - 3x - 4}$

Since this is a rational expression, we must not let the denominator equal zero. What values of x make the denominator zero?

$x^2 - 3x - 4 = 0$
$(x - 4)(x + 1) = 0$
$x = 4$ or $x = -1$

Thus all real numbers work except -1 and 4. In interval notation this is written:

$(-\infty, -1) \cup (1, 4) \cup (4, \infty)$

(B) $f(x) = \sqrt{2x + 3} - 1$

Since this function involves a square root we must make sure the radicand is non-negative:

$2x + 3 \geq 0$
$2x \geq -3$
$x \geq -3/2$

Thus the domain is all real numbers greater than or equal to -3/2. In interval notation this is written: $[-3/2, \infty)$

(C) $f(x) = x^2 + 5x - 7$

This function does not involve a rational expression or a square root so the domain is all real numbers. In interval notation this is written: $(-\infty, \infty)$

(D) $f(x) = \dfrac{2}{\sqrt{x - 1}}$

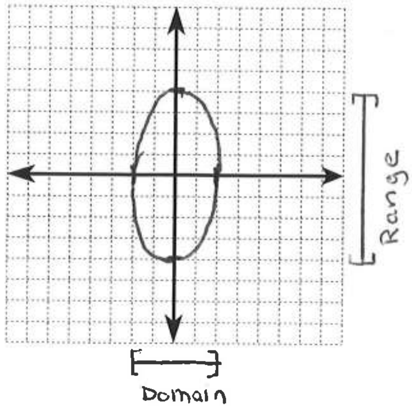
We must be very careful with this function since it involves both a rational expression and a square root. The square root requires the radicand to be greater than or equal to zero, that is, $x - 1 \geq 0$. However, since the square root is in the denominator and we can not divide by zero, we cannot let $x - 1 = 0$. Thus,

$x - 1 > 0$
$x > 1$

In interval notation this is written: $(1, \infty)$

Now let's look at some graphs and determine their domain and range.

(A)



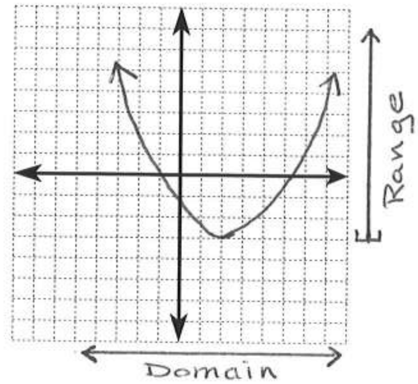
From the graph we see the x-values extend from -2 to 2 inclusive, so

Domain: [-2, 2]

The y-values extend from -4 to 4 inclusive, so

Range: [-4, 4]

(B)



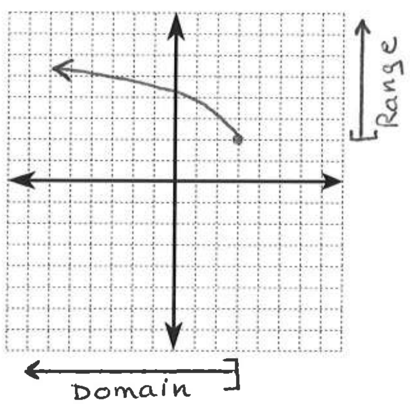
From the graph we see the x-values extend indefinitely in both directions, so

Domain: $(-\infty, \infty)$

The y-values start at -3 and keep going up, so,

Range: $[-3, \infty)$

(C)



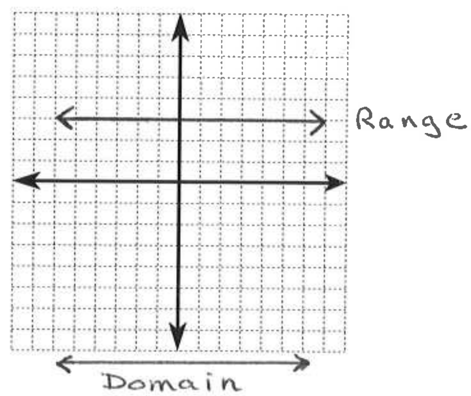
From the graph we see the x-values extend indefinitely on the left, but stop at 3, so

Domain: $(-\infty, 3]$

The y-values start at 2 and keep going up, so

Range: $[2, \infty)$

(D)



From the graph we see the x-values extend indefinitely in both directions, so

Domain: $(-\infty, \infty)$

The only y-value is 3, so

Range: {3} notice this is not an interval

MAC 1105

Domain & Range

Determine the domain and range of the relation.

1) {(11, −6), (−9, 4), (1, 3), (−1, 2), (8, −5)}

Find the domain of the function.

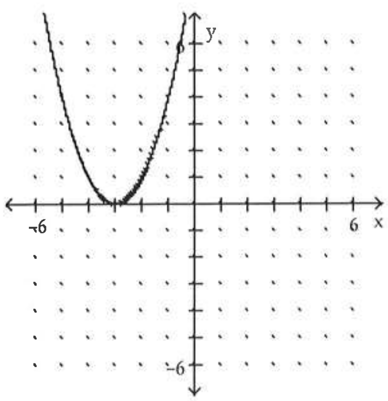
2) $f(x) = 5x^2 + 3x - 1$

3) $f(x) = \sqrt{x-6}+2$

4) $f(x) = \dfrac{1}{x^2 + 3x - 10}$

Find the domain and range of the relation. Assume the ends of the relation continue on.

5)



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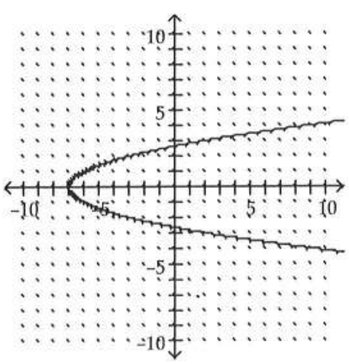

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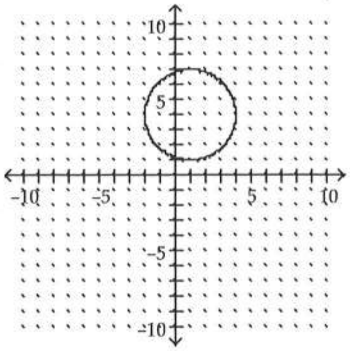

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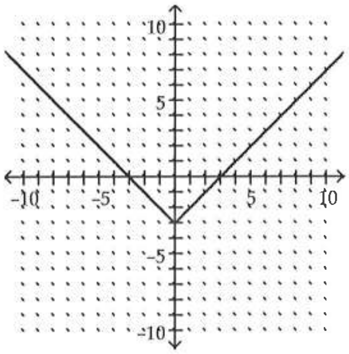

Answer Key

1) Answer: $D = \{8, 11, 1, -9, -1\}$; $R = \{-5, -6, 3, 4, 2\}$
2) Answer: $(-\infty, \infty)$
3) Answer: $[6, \infty)$
4) Answer: $(-\infty, -5) \cup (-5, 2) \cup (2, \infty)$
5) Answer: domain $(-\infty, \infty)$; range $[0, \infty)$
6) Answer: domain $[-7, \infty)$; range $(-\infty, \infty)$
7) Answer: domain $[-2, 4]$; range $[1, 7]$
8) Answer: domain $(-\infty, \infty)$; range $[-3, \infty)$