Slide 1

3.1 Statements and Logical Connectives

This section introduces the basics of symbolic logic, which uses letters to represent statements, and symbols for words such as "*and*", "*or*", "*not*", and "*if — then*".

A __________________ is a sentence that is either true or false. The following are examples of statements:

- Alaska is geographically the largest state in the U.S. (true)
- 2 + 1 = 6. (false)
- February has 30 days. (false)

A **simple statement** conveys a single idea. (See previous examples).

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The following sentences are **NOT** statements:

- Florida is the best state. (subjective)

- Is it raining? (a question)

- I am lying to you. (Neither true nor false – paradox)

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Negations

The negation of the statement "I have a red car" is the statement "I do not have a red car." The negation of a true statement is false, and the negation of a false statement is true.

The negation is symbolized by ~ and is read "not."

Example Write the negation of

(A) Florida has a governor.

(B) The sun is not a star.

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Quantifiers are words to indicate how many cases of a particular situation exist:

Special care must be used when negating statements containing quantifiers.

What is the negation of "Some dogs have fleas"?

Some means *at least one* dog has fleas.

If this is a true statement, the negation must be false.

The negation is ______________________

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What is the negation of "All plants have a flower"?

This statement is false since a fern is a plant, but it does not have a flower. Since this statement is false, the negation must be true.

We might be tempted to say *"No plants have a flower."* However, this statement is also false (since a rose is a plant and it has a flower).

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The correct negation of "All plants have a flower" is

"Not all plants have a flower." OR

"At least one plant does not have a flower." OR

____

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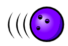
The Negation of Quantified Statements

(see page 85)

All are ↔	**Some are not**
None are ↔	**Some are**

Example Write the negation of the statement.

#24 No money grows on trees. 

#26 All bowling balls are round. 

#34 Some people who earn money do not pay taxes.

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A **compound statement** may be formed by combining two or more simple statements. Various _______________ (and, or, not, if — then) can be used in forming compound statements. It is common to represent each simple statement with a lowercase letter and each connective with a symbol.

AND Statements (conjunction) Symbolized by $\wedge$

Example Write the conjunction in symbolic form.

p: Chris collects coins  

q: Jack is a catcher

"Chris collects coins and Jack is not a catcher."

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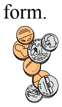
Other words sometimes used to express a conjunction are *but*, *however*, or *nevertheless*.

OR Statements (disjunction) Symbolized by $\vee$

In this course we will use the *inclusive or*.

Example Write the disjunction in symbolic form.

p: Chris collects coins  

q: Jack is a catcher

"Chris does not collect coins or Jack is not a catcher."

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When a compound statement contains more than one connective, a comma is used to indicate which simple statements are to be grouped together.

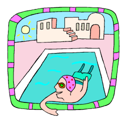
The simple statements on the same side of the comma are to be grouped together within parentheses.

<u>Example</u> #68 Write in words $(p \lor q) \land \sim r$

p: The water is 70?.

q: The sun is shining.

r: We go swimming.



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If — Then Statements (Conditional) Symbolized by $\rightarrow$

$p \rightarrow q$ is read "if p, then q."

antecedent consequent

<u>Example</u> #42 Write in symbolic form

p: The charcoal is hot

q: The chicken is on the grill

"If the chicken is not on the grill, then the charcoal is not hot."

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A negation only negates the statement right after it. To negate a compound statement, you must use parentheses. When a negation symbol is in front of a statement in parentheses it negates the entire statement in parentheses and is usually read *"It is not true that…"* or *"It is false that…"*

<u>Example</u> #64 Write in symbolic form

"It is false that if the apartment is hot then the air conditioner is not working."

p: The temperature is 90°.

q: The air conditioner is working.

r: The apartment is hot.

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If and Only If Statements (Biconditional) Symbolized by $\leftrightarrow$

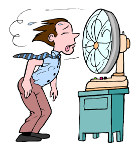
$p \leftrightarrow q$ is read "_______________________"

Example #62 Write in symbolic form.

p: The temperature is 90?.

q: The air conditioner is working

r: The apartment is hot.



"The temperature is not 90? if and only if the air conditioner is not working, or the apartment is not hot.

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Summary of connectives on page 90

Dominance of Connectives

Just like we have an order of operations in arithmetic, there is an order in which we evaluate connectives.

Least dominate (evaluate first)

1. Negation
2. Conjunction, Disjunction
3. Conditional
4. Biconditional

Most dominate (evaluate last)

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Example

(A) Add parentheses by using the dominance of connectives

(B) indicate whether the statement is a negation, conjunction, disjunction, conditional, or biconditional.

#82 $\sim p \wedge r \leftrightarrow \sim q$

#86 $q \rightarrow p \wedge \sim r$

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3.2 Truth Tables for Negation, Conjunction and Disjunction

We use a **truth table** to determine when a compound statement is true or false.

If a compound statement consists of two simple statements, the truth table will have 4 possible cases:

p	q

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Conjunction Truth Table

Consider the following statements:

p: Today is Monday

q: I have a math class

In everyday language, ***and*** implies the idea of "both."

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

$p \wedge q$ is true only when both p and q are true.

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Disjunction Truth Table

Consider the following statements:

p: Today is Monday

q: I have a math class

or implies the idea of p or q, or "both."

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

$p \vee q$ is true when either p is true, q is true, or both p and q are true.

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A General Format for Constructing Truth Tables

p q Compound Statement

Example Construct a truth table for the statement.

#8 $p \wedge \sim q$

#10 $\sim(p \vee \sim q)$

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A truth table for a compound statement with 3 simple statements.

Example Construct a truth table for the statement.

#18 $\sim p \wedge (q \vee r)$

In general, the number of distinct cases (rows) in a truth table with n distinct simple statements is _________.

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If we want to find the truth value of a compound statement for a specific case (we know the truth values of the simple statements), we do not have to create the entire truth table.

Example Determine the truth value of the statement.

#36 p is true q is false r is true

$(p \vee \sim q) \wedge \sim(p \wedge \sim r)$

#44 9 – 6 = 15 and 7 – 4 = 3

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3.3 Truth Tables for the Conditional and Biconditional

Consider the conditional statement

"If I am elected, then I will lower taxes."

Under what conditions will this statement be true?

Case I (T→T): I am elected and taxes go down. ________

Case II (T→F): I am elected and taxes do not go down. _______

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Case III (F→T): I am not elected and taxes go down. ________

Case IV (F→F): I am not elected and taxes do not go down. ____________

P	Q	P→Q
T	T	
T	F	
F	T	
F	F	

p→q is true in every case except when p is true and q is false.

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Example Construct a truth table for the statement.

#8 $\sim q \rightarrow \sim p$

#18 $r \wedge (\sim q \rightarrow p)$

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Biconditional Statement

p ↔ q means that p → q and q → p

p	q	p → q	q → p	(p → q) ∧ (q → p)
T	T			
T	F			
F	T			
F	F			

p ↔ q is true only when both p and q have the same truth value (both true or both false).

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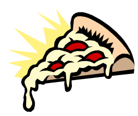
Example #28 Write the statement in symbolic form. Then construct a truth table.

"The pizza will be delivered if and only if the driver finds the house, or the pizza will not be hot."



Example #48 Find the truth value if

p is true, q is false r is true

p ↔ (~q ∧ r)

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Tautologies and Self-contradictions

If a compound statement is always false it is called a **self-contradiction.**

If a compound statement is always true it is called a **tautology**.

Example Determine whether the statement is a tautology, a self-contradiction, or neither.

(p ? q) ? (~p ∨ q)

An **implication** is a conditional statement that is a tautology.

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3.4 Equivalent Statements

Two statements are **equivalent**, symbolized $\Leftrightarrow$, if both statements have exactly the same truth values in the answer columns of the truth tables.

Example Determine if the statements are equivalent.

$\sim p \wedge \sim q$ and $\sim(p \vee q)$

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DeMorgan's Law

$\sim(p \wedge q) \Leftrightarrow$

$\sim(p \vee q) \Leftrightarrow$

Example Use DeMorgan's laws to determine whether the two statements are equivalent.

$\sim(\sim p \wedge q)$ and $p \wedge \sim q$

Example #32 Use DeMorgan's laws to write an equivalent statement for the sentence.

It is false that the ink is red and the pen has a ball point.

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Does $p \rightarrow q$ have an equivalent statement?

Construct a truth table for $p \rightarrow q$ and $\sim p \vee q$

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$
T	T			
T	F			
F	T			
F	F			

$p \rightarrow q$ and $\sim p \vee q$ are equivalent statements

This allows us to write a conditional statement as a disjunction or a disjunction as a conditional statement.

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To change a conditional statement to a disjunction, negate the antecedent, change the conditional symbol to a disjunction symbol, and keep the consequent the same.

To change a disjunction statement to a conditional statement, negate the first statement, change the disjunction symbol to a conditional symbol, and keep the second statement the same.

Example Write an equivalent form of the statement.

#40 The roller-coaster ride was exciting or the roller-coaster ride was out of service.

#42 If we do not renew the subscription, then the magazine will stop coming.

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Recall, $p \leftrightarrow q$ means that $(p \to q) \wedge (q \to p)$. Thus

$p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are equivalent statements.

Example Write the statement in an equivalent form.

#48 You need to pay taxes if and only if you receive income.

Negation of the Conditional Statement.

Example Find an equivalent statement for $\sim(p \to q)$.

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Variations of the conditional statement, $p \to q$

- **Converse** of the conditional: $q \to p$
- **Inverse** of the conditional: $\sim p \to \sim q$
- **Contrapositive** of the conditional $\sim q \to \sim p$

p	q	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T				
T	F				
F	T				
F	F				

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From the truth table we see the ***conditional*** statement is equivalent to the ***contrapositive*** statement,

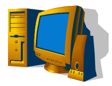
$p \rightarrow q \;\Leftrightarrow$ ________________

Also, the ***converse*** statement is equivalent to the ***inverse*** statement,

$q \rightarrow p \;\Leftrightarrow$ _______________

Example Write the converse, inverse, and contrapositive of the statement:

#50 If the computer goes on sale, then we will buy the computer.



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Example Determine which, if any, of the three statements are equivalent.

#66

a) If today is Monday, then tomorrow is not Wednesday.

b) It is false that today is Monday and tomorrow is not Wednesday.

c) Today is not Monday, or tomorrow is Wednesday.

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3.5 Symbolic Arguments

A **symbolic argument** consists of a set of premises and a conclusion. We generally write it in symbolic form to determine its validity.

An argument is ____________ when its conclusion necessarily follows from a given set of premises.

An argument is ____________ **(fallacy)** when the conclusion does not necessarily follow from the given set of premises.

When an argument is valid, the conclusion must follow from the premises. It is not necessary for the premises or the conclusion to be true statements.

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Consider the following argument,

If John has a B in math, he will graduate. Premise 1

John does have a B in math. Premise 2

Therefore John will graduate. Conclusion

In symbolic form,

p: John has a B in math

q: John will graduate

$p \rightarrow q$

p

$\therefore q$

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To see if the argument is valid, construct a truth table for

[(premise 1) $\wedge$ (premise 2)] $\rightarrow$ conclusion

If the truth table answer column is true in every case (a tautology) then the argument is valid.

$[(p \rightarrow q) \wedge p] \rightarrow q$

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$[(p \rightarrow q) \wedge p] \rightarrow q$
T	T			
T	F			
F	T			
F	F			

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$p \rightarrow q$

p

$\therefore q$

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Procedure to Determine Whether an Argument is Valid (p. 129)

1. Write the argument in symbolic form.

2. Compare the form of the argument with forms that are known to be valid or invalid. If there are no known forms to compare it with, or you do not remember the forms go to step 3.

3. Write the argument in symbolic form:

   $[(\text{premise 1}) \wedge (\text{premise 2})] \rightarrow \text{Conclusion}$

4. Construct a truth table for the statement in step 3.

5. If the answer column of the truth table has all trues (tautology) the argument is valid, otherwise it is invalid.

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Example Determine the validity of the argument

If the butler is guilty of the crime, then his shoes will be muddy.

The butler's shoes are muddy.

Therefore the butler is guilty.

p: butler guilty of the crime

q: butler's shoes are muddy

$p \rightarrow q$

q

$\therefore\ p$

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p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T			
T	F			
F	T			
F	F			

$p \rightarrow q$

q

$\therefore\ p$

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Example Determine the validity of the argument

Either elephants are blue or monkeys are green.

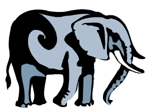
Elephants are grey (not blue).

Therefore monkeys are green.

p: elephants are blue

q: monkeys are green

$$\begin{array}{l} p \lor q \\ \sim p \\ \hline \therefore\ q \end{array}$$



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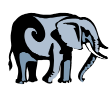
p	q	$p \lor q$	$(p \lor q) \land \sim p$	$[(p \lor q) \land \sim p] \to q$
T	T			
T	F			
F	T			
F	F			

$$\begin{array}{l} p \lor q \\ \sim p \\ \hline \therefore\ q \end{array}$$



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Standard Forms of Arguments (p. 131)

Valid Arguments

Law of Detachment

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore\ q \end{array}$$

Law of Contraposition

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore\ \sim p \end{array}$$

Law of Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

Disjunctive Syllogism

$$\begin{array}{l} p \lor q \\ \sim p \\ \hline \therefore\ q \end{array}$$

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Invalid Arguments

Fallacy of the Converse

$p \rightarrow q$

q

$\therefore\ p$

Fallacy of the Inverse

$p \rightarrow q$

$\sim p$

$\therefore\ \sim q$

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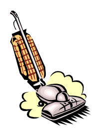
Example Determine if the argument is valid.

#38 If you cook the meal, then I will vacuum the rug.

I will not vacuum the rug.

∴ You will not cook the meal.

#40 It is snowing and I am going skiing.

If I am going skiing, then I will wear a coat.

∴ If it is snowing, then I will wear a coat.

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3.6 Euler Diagrams & Syllogistic Arguments

In the last section we studied symbolic arguments which used the connectives *and, or, not, if -- then*, and *if and only if.*

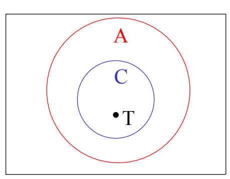
In this section, we will study syllogistic arguments which use the quantifiers *all, some,* and *none.* We will use an **Euler Diagram** to visualize the argument.

Example Determine if the argument is valid.

All cats are animals.

Tom is a cat.

∴ Tom is an animal.

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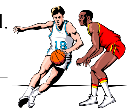
Example Use an Euler diagram to determine whether the syllogism is valid or invalid.

#10 All As are Bs.
All Bs are Cs.
∴ All As are Cs.

#14 No basketball players are greater than 8 ft tall.
Pete is not a basketball player.
∴ Pete is greater than 8 ft tall.



Whenever testing the validity of an argument, always try to show that the argument is invalid.

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#16 Some politicians are stuffy.
Todd Hall is a politician.
∴Todd Hall is not stuffy.

#22 Some desks are made of wood.
All paper is made of wood.
∴Some desks are made of paper.

All ballplayers are athletes.
All athletes are healthy.
Sheila is healthy.
∴ Sheila is a ballplayer.